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Permitted Announcement Logic with Unary Modal Operators*

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Abstract. Based on a formal language of permitted announcements advanced by Balbiani and Seban, we propose a formalization of "it is permitted to announce something" with unary operators. To do it, we cut off the unary fragment of the language and give it neighborhood semantics. We investigate the definability of unary operators, two different types of update methods, and reduction theorems in this language. We also axiomatize the logic. As an application, we formalize some norms of assertion and the Moorean sentences in the language.

Keywords: Public announcement · Permission · Neighborhood semantics · Update neighborhood models · Reduction theorems · Norms of assertion

1 Introduction

G. E. Moore [5] noticed that it is odd to say this sentence: "It is raining, but I do not believe that it is raining." Hintikka [2] found another sentence, "it is raining, but I do not know that it is raining" is also odd to say. If we use $K_i p$ and $B_i p$ to express "agent i knows/believes p", we can formalize those sentences as $\neg K_i p \wedge p$ and $\neg B_i p \wedge p$. We call a sentence that has one of these forms a Moorean sentence. In [13], Williamson claims that the oddity of Moorean sentences can be explained by violating some assertion norms, which means that the agent is not permitted to say Moorean Sentences under those norms.

This essay focus on the formalization of permitted announcements. For example, we want to express "it is not permitted to announce Moorean Sentences" and "if someone is permitted to announce φ, then she knows φ" in a formal logic language.

An obvious starting point is the logic of public announcements (PAL), which was famously proposed by Plaza [7]. The language of PAL can express "after announcing ψ, φ is true" by $[\psi]\varphi$ through an update on epistemic models. A detailed studies can be found in [10]. But this language can not express the deontic standards we want, since PAL can not express the permission of announcements.

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Due to this limitation, Balbiani and Seban [1] proposed a formal language POPAL (permitted and obligated announcement logic, noted by $\mathcal{L}_{popal}$) by adding $P(\psi,\varphi)$ and $O(\psi,\varphi)$ to the language of PAL. $P(\psi,\varphi)$ and $O(\psi,\varphi)$ means that "ψ is true, and after announcing ψ, it is permitted/obligated to announce φ". Therefore, $\mathcal{L}_{popal}$ has the power to formalize some deontic announcement sentences. For $\mathcal{L}_{popal}$, Balbiani and Seban use a neighborhood-like semantics to interpret $\mathcal{L}_{popal}$, and they also established a sound and complete axiomatization POPAL w.r.t all models.

This language seems good enough for our aims, but as some relevant studies developed, there are some further works we can do on permitted announcements:

First, Ma and Sano [3] studied two ways of updating neighborhood models. The semantics of $\mathcal{L}_{popal}$ only use the first kind of update. By referring to this recent study, we give a full picture of how different update methods influence the validities of the reduction theorems. We will discuss these two kinds of update methods and give a modified update method for the announcement operator.

Second, Seban and Ditmarsch further consider the expressive power of $\mathcal{L}_{popal}$ in [11]. They proved $\mathcal{L}_{popal}$ is expressively equal to its unary fragment. But they do not give the axiomatization of the fragment. This unary fragment is worth further consideration. We will give a new neighborhood semantics to it and discuss the axiomatization of the fragment under neighborhood semantics.

In a nutshell, this essay aims at introducing $\mathcal{L}_{upopal}$ by trying two kinds of update methods. In the rest of the essay, first in Section 2, we will review the $\mathcal{L}_{popal}$ (2.1), and define the unary fragment of it (2.2). In Section 3, we will establish neighborhood semantics for $\mathcal{L}_{upopal}$ (3.1) and introduce a technically useful update method for $\mathcal{L}_{upopal}$ (3.2). In Section 4, we will give a sound and complete axiomatization UPOPAL of $\mathcal{L}_{upopal}$ (4.1-4.3). In Section 5, we will formalize knowledge and belief norms of assertion as an example of UPOPAL (5.1) and show that Moorean sentences are never permitted to announce under those norms (5.2).

2 From $\mathcal{L}_{popal}$ to its Unary Fragment

2.1 A Review of $\mathcal{L}_{popal}$

This part we briefly present the work of Balbiani and Seban [1]. The logic of permitted and obligated announcements is an extension of the multi-agents epistemic logic of public announcements. Given a countable set of agents N and a countable set of propositional atoms Θ, the language $\mathcal{L}_{popal}$ of *Permission and Obligation Public Announcement Logic* (POPAL) is defined as follow:

$$\varphi ::= \ p \mid \bot \mid \neg\varphi \mid \varphi \vee \varphi \mid K_i\varphi \mid [\varphi]\varphi \mid P(\varphi,\varphi) \mid O(\varphi,\varphi)$$

where $p \in \Theta$, $i \in N$. $\top$, $\varphi \wedge \psi$, $\varphi \to \psi$ and $\langle\psi\rangle\varphi$ are defined as the abbreviation of $\neg\bot$, $\neg\varphi \vee \neg\psi$, $\neg\varphi \vee \psi$ and $\neg[\psi]\neg\varphi$. The intuitive reading of K_ip is "agent i knows that p is true" whereas $[\psi]\varphi$ is read as "after announcing ψ, φ is true". $P(\psi, \varphi)$ and $O(\psi, \varphi)$ are read as "ψ is true and after announcing ψ, it is permitted/obligated to announce φ".

A model over non-empty sets Θ and N is a tuple $\langle S, \sim_i, V, \mathcal{P}\rangle$, where S is a non-empty set of states, for each $i \in N$, $\sim_i$ an epistemic equivalence relation on S, V a valuation function from Θ to subsets of S, and $\mathcal{P}$ a neighborhood relation that $\mathcal{P} \subseteq S \times 2^S \times 2^S$, if $(s, S', S'') \in \mathcal{P}$, then $s \in S'' \subseteq S'$. The semantics of announcement operator and new operators O/P are defined as:

- $\mathcal{M}, s \vDash [\psi]\varphi$ iff $\mathcal{M}, s \vDash \psi \Rightarrow \mathcal{M}_\psi, s \vDash \varphi$.
- $\mathcal{M}, s \vDash P(\psi, \varphi)$ iff for some $(s, [\![\psi]\!]_\mathcal{M}, S'') \in \mathcal{P}$, $S'' \subseteq [\![\langle\psi\rangle\varphi]\!]_\mathcal{M}$.
- $\mathcal{M}, s \vDash O(\psi, \varphi)$ iff for all $(s, [\![\psi]\!]_\mathcal{M}, S'') \in \mathcal{P}$, $S'' \subseteq [\![\langle\psi\rangle\varphi]\!]_\mathcal{M}$.

where $\mathcal{M}_\psi$ is an update model and defined as $(S_\psi, \sim_i^\psi, V_\psi, \mathcal{P}_\psi)$: $S_\psi = [\![\psi]\!]_\mathcal{M}$; and for all i, $\sim_i^\psi = \sim_i \cap (S_\psi \times S_\psi)$; and for all $p \in \Theta$, $V_\psi(p) = V(p) \cap S_\psi$; and $\mathcal{P}_\psi = \{(s, S', S'') \in \mathcal{P} \mid s \in S_\psi, S' \subseteq S_\psi, S'' \subseteq S_\psi\}$. The update method used here is a limitation on the original model. The limitation of announcement ψ throw all the $\mathcal{P}$ members which are not contained by $[\![\psi]\!]_\mathcal{M}$. In [3], Ma and Sano named this way Subset-update. We will introduce another update method in Section 3.

2.2 Cut off the Unary Fragment

We focus on the unary fragment of $\mathcal{L}_{upopal}$. For all formulas $\chi \in \mathcal{L}_{popal}$, we write $P\chi \ := \ P(\top, \chi)$ and $O\chi \ := \ O(\top, \chi)$. We name the fragment of $\mathcal{L}_{popal}$ with unary operators $\mathcal{L}_{upopal}$. From [11] We have:

Proposition 1. $\vDash \langle\psi\rangle P(\top, \varphi) \leftrightarrow P(\psi, \varphi)$ *and* $\vDash \langle\psi\rangle O(\top, \varphi) \leftrightarrow O(\psi, \varphi)$

Proof. See [11] for details.

In semantics, we have:

- $\mathcal{M}, s \vDash P\varphi$ iff for some $(s, S', S'') \in \mathcal{P}$, $S'' \subseteq [\![\varphi]\!]_\mathcal{M}$.
- $\mathcal{M}, s \vDash O\varphi$ iff for all $(s, S', S'') \in \mathcal{P}$, $S'' \subseteq [\![\varphi]\!]_\mathcal{M}$.

Two important observations: (1) Proposition 1 tells us we can rewrite the $\mathcal{L}_{popal}$ into a simplified unary version without losing expressive power. (2) From the semantics of $P\varphi$ and $O\varphi$, we can find that their satisfaction does not rely on S'. Combining these two observations, we find that it is possible to give neighborhood semantics to $\mathcal{L}_{upopal}$

3 Semantics and Update Methods

3.1 A Neighborhood Semantics of $\mathcal{L}_{upopal}$

Definition 1. *N and Θ are defined in the same way as $\mathcal{L}_{popal}$. A model $\mathcal{M}$ is a tuple $\langle S, \{\sim_i\}_{i \in AG}, V, N\rangle$, where S is a non-empty set of states, $\sim_i$ is an epistemic equivalence relation on S, and V is a valuation function from Θ to subsets of S. N is a neighborhood function that $N : S \to \mathcal{P}(\mathcal{P}(S))$. Let $N(s) \neq \emptyset$.*

Using the method provided by [3], we define two different ways to update our models:

Definition 2. *For any model $\mathcal{M}$ and any $\psi \in \mathcal{L}_{upopal}$, the restricted model of $\mathcal{M}^{*\varphi} = \langle [\![\varphi]\!]_{\mathcal{M}}, \sim_i^{*\varphi}, V^{*\varphi}, N^{*\varphi}\rangle$ for $* \in \{\subseteq, \Cap\}$, where for all $p \in \Theta$ and $i \in AG$: $\sim_i^{*\varphi} = \sim_i \cap([\![\varphi]\!]_{\mathcal{M}} \times [\![\varphi]\!]_{\mathcal{M}})$, $V^{*\varphi}(p) = V(p) \cap [\![\varphi]\!]_{\mathcal{M}}$. $N^{*\varphi}$ is defined as:*

$$N^{\subseteq\varphi}(s) = \{Y \mid Y \subseteq [\![\varphi]\!]_{\mathcal{M}} \wedge Y \in N(s)\}.$$

$$N^{\Cap\varphi}(s) = \{Y \mid \exists Z((Z \in N(s)) \wedge (\emptyset \neq Y = Z \cap [\![\varphi]\!]_{\mathcal{M}})\}.$$

Notice that in $N^{\Cap\varphi}(w)$, we must warrant every points w in $\mathcal{M}^{\Cap\varphi}$ has no empty neighbor subset.

Definition 3. *Let $\mathcal{M}$ be a model and s be a state of S, we define:*

- *$\mathcal{M}, s \vDash p$ iff $w \in V(p)$;*
- *$\mathcal{M}, s \nvDash \bot$;*
- *$\mathcal{M}, s \vDash \neg\varphi$ iff $\mathcal{M}, s \nvDash \varphi$;*
- *$\mathcal{M}, s \vDash \varphi \vee \psi$ iff $\mathcal{M}, s \vDash \varphi$ or $\mathcal{M}, w \vDash \psi$;*
- *$\mathcal{M}, s \vDash K_i\varphi$ iff for every s with $s \sim_i s'$, $\mathcal{M}, s' \vDash \varphi$;*
- *$\mathcal{M}, s \vDash [\psi]\varphi$ iff $\mathcal{M}, s \vDash \psi \Rightarrow \mathcal{M}^{*\psi}, s \vDash \varphi$.*
- *$\mathcal{M}, s \vDash P\varphi$ iff for some $X \in N(s)$, $X \subseteq [\![\varphi]\!]_{\mathcal{M}}$.*
- *$\mathcal{M}, s \vDash O\varphi$ iff for all $X \in N(s)$, $X \subseteq [\![\varphi]\!]_{\mathcal{M}}$.*

For all $\varphi \in \mathcal{L}_{upopal}$, $\mathcal{M} \vDash \varphi$ iff for all $s \in S$, $\mathcal{M}, s \vDash \varphi$, and $\vDash \varphi$ iff for all models $\mathcal{M}$ we have $\mathcal{M} \vDash \varphi$.

Now we discuss the interpretation of our semantics. The intuitive reading of S is all the states we consider possible. $\sim_i$ denotes the epistemic indistinguishable states of agent i. We define a "knowledge cluster" for every $s \in S$ and every $i \in AG$ by $[s]_{\sim_i}$, where $[s]_{\sim_i}$ is the equivalence class of $\sim_i$ on s. $\mathcal{M}, s \vDash K_i\varphi$ if and only if φ is satisfied in every points of $[s]_{\sim_i}$. V is a function distribute every atom proposition a subset.

The interpretation of $N(s)$ needs more explanation. $N(s)$ collect some subsets of S for s. We can see every $N(s)$ as an "ideology set" of s, which collect every "ideologies" of s. If an announcement φ does not contradict some ideologies, then $P\varphi$ is satisfied in the state, which means "it is permitted to announce". Although, it may contradict some other ideologies. If an announcement does not contradict any ideologies, then $O\varphi$ is satisfied in the state. But in this semantics, it does not mean "it is obligated to announce φ" as in $\mathcal{L}_{popal}$, it only means there is no deontic restriction to stop announcing φ. So, the interpretation of $O\varphi$ is "it is free to announce φ." Using terms in [6], $P\varphi$ is a $\langle\ ]$ operator, which means for some subset, every point in the subset satisfies some formulas. $O\varphi$ is a $[\ ]$ operator, which means for all subset, every point in the subset satisfy some formulas. (The notation $\langle$ and $\rangle$ stand for *exists*, $[$ and $]$ stand for *for all*.)

We can use ideology to explain two update methods given by Definition 4. Subset-update means, that for every ideology that contradicts our announcement, we throw the whole ideology away in update models. Intersection-update means, that for every ideology that contradicts our announcement, we modify the ideology by cutting the contradictory part in update models.

Notice that $O\varphi \leftrightarrow \neg P\neg\varphi$ is not something we want. If someone is "free to assert p", it does not only mean she is "not permitted to announce not p", she is also not permitted to announce something that implies not p. Similarly, if someone is "not permitted to announce not p", it does not only mean she is "free to assert p", but she can also "not announce anything". We will see in Proposition 4 that operators O and P are not mutually definable.

We should also pay attention to the condition that every member in $N(s)$ can not be an empty set. This condition is important for the proof of completeness. It also preserves that $P\bot$ is impossible.

We continue with two model invariance results in weaker languages, which is important for our later work on reduction theorems. An easy obervation is the satisfaction of $P\varphi$ is preserved under adding bigger subset and the satisfaction of $O\phi$ is preserved under adding smaller subset. A precise expression is:

Proposition 2. *Let $\mathcal{L}_{upopal-O-[\ ]}$ be $\mathcal{L}_{upopal}$ without O and announcement operator, and let $\mathcal{L}_{upopal-P-[\ ]}$ without P and announcement operator. Consider two pointed models $\mathcal{M}, s$ and $\mathcal{M}', s'$. We have:*

- *If $N(s) \subseteq N'(s)$, and for any $X' \in N'(s)$, exists a $X \in N(s)$, where $X \subseteq X'$, then we have for any $\varphi \in \mathcal{L}_{upopal-O-[\ ]}$, $M, s \vDash \varphi$ iff $M', s \vDash \varphi$.*
- *If $N(s) \subseteq N'(s)$, and for any $X' \in N'(s)$, exists a $X \in N(s)$, where $\emptyset \neq X' \subseteq X$, then we have for any $\varphi \in \mathcal{L}_{upopal-P-[\ ]}$, $M, s \vDash \varphi$ iff $M', s \vDash \varphi$.*

Proof. The proof is obvious. We left it to readers.

Can we find a structural property preserve invariance under a richer language $\mathcal{L}_{upopal-[\ ]}$? Yes. From Proposition 6, we know when we add or delete some non-important neighbor subsets, modal satisfaction is invariant. So in $\mathcal{L}_{upopal-[\ ]}$, what we should do is to find some non-important subsets for both P and O:

Proposition 3. *If $N(s) \subseteq N'(s)$, for any $X' \in N'(s)$, exists a $X \in N(s)$, where $X \subseteq X'$, and exists a $Y \in N(s)$, where $\emptyset \neq X' \subseteq Y$, then we have for any $\varphi \in \mathcal{L}_{upopal-[\ ]}$, $M, s \vDash \varphi$ iff $M', s \vDash \varphi$.*

Proof. This is proved straight from Proposition 2.

Using this result, we can prove a proposition about the expressive power of P/O operators:

Proposition 4. *Operators P and O are not mutually definable in $\mathcal{L}_{upopal-[\,]}$.*

Proof. Assume operators P and O are mutually definable, then $\mathcal{L}_{upopal-P-[\,]}$ is expressively equal to $\mathcal{L}_{upopal-O-[\,]}$. Now we should prove $\mathcal{L}_{upopal-P-[\,]}$ is less expressive than $\mathcal{L}_{upopal-O-[\,]}$ and vice versa.

Consider a model $\mathcal{M} = \langle S, \sim_i, V, N\rangle$, where $S = \{s_1, s_2, s_3\}$, $V(p) = \{s_1, s_2\}$, $N(s_1) = \{\{s_1, s_2\}\}$.

For contradiction, assume for any formula φ, $\varphi \in \mathcal{L}_{upopal-P-[\,]}$, exists a $\psi \in \mathcal{L}_{upopal-O-[\,]}$, where $\vDash \psi \leftrightarrow \varphi$. Let $\vDash Op \leftrightarrow \chi$, $\chi \in \mathcal{L}_{upopal-O-[\,]}$. Let $\mathcal{M}' = \langle S, \sim_i, V, N'\rangle$ where $N'(s_1) = \{\{s_1, s_2\}, \{s_1, s_2, s_3\})\}$. From Proposition 6, for all formulas φ in $\mathcal{L}_{upopal-O-[\,]}$, $\mathcal{M}, s_1 \vDash \varphi$ iff $\mathcal{M}', s_1 \vDash \varphi$. But $\mathcal{M}, s_1 \vDash Op$ and $\mathcal{M}', s_1 \nvDash Op$, then $\mathcal{M}, s_1 \vDash \chi$ and $\mathcal{M}', s_1 \nvDash \chi$. This is a contradiction.

Similarly, let $\vDash Pp \leftrightarrow \chi$, $\chi \in \mathcal{L}_{upopal-P-[\,]}$. Let $\mathcal{M}'' = \langle S, \sim_i, V, N''\rangle$ and $N''(s_1) = \{\{s_1\}, \{s_1, s_2\}\}$. For all formulas φ in $\mathcal{L}_{upopal-P-[\,]}$, $\mathcal{M}, s_1 \vDash \varphi$ iff $\mathcal{M}', s_1 \vDash \varphi$. But $\mathcal{M}, s_1 \vDash Pp$ and $\mathcal{M}'', s_1 \nvDash Pp$, then $\mathcal{M}, s_1 \vDash \chi$ and $\mathcal{M}', s_1 \nvDash \chi$. This is a contradiction.

3.2 Dealing with Reduction Theorems

Reduction theorems are used to translate a language with announcement operators into a language without announcement operators. The expressive power of these two languages is equal.

Proposition 5. *For all $p \in \Theta$, all φ, $\psi \in \mathcal{L}_{upopal}$, for both update semantics, we have:*

- $\vDash [\psi]p \leftrightarrow p$;
- $\vDash [\psi]\neg\varphi \leftrightarrow \neg[\psi]\varphi$;
- $\vDash [\psi](\varphi \vee \chi) \leftrightarrow ([\psi]\varphi \wedge [\psi]\chi)$;
- $\vDash [\psi]K_i\varphi \leftrightarrow K[\psi]\varphi$;
- $\vDash [\psi][\chi]\varphi \leftrightarrow [\langle\psi\rangle\chi]\varphi$.

Proof. These are standard results, see [10] for details.

But, we do not have reduction theorems in $\Cap$ semantics for P and we have no reduction theorems in $\subseteq$ semantics for O.

Proposition 6. *For all φ, $\psi \in \mathcal{L}_{upopal}$:*

- $N^{\subseteq\psi}$:
  - $\vDash [\psi]\varphi \leftrightarrow (\psi \rightarrow P(\langle\psi\rangle\varphi))$;
  - *No reduction theorems for $[\psi]O\varphi$ in $\mathcal{L}_{upopal}$;*

- $N^{\Cap\psi}$:
  - *No reduction theorems for* $[\psi]P\varphi$ *in* $\mathcal{L}_{upopal}$;
  - $\vDash [\psi]O\varphi \leftrightarrow (\psi \to O([\psi]\varphi))$;

Proof. We only consider the case where there are no reduction theorems. For all $\mathcal{M}$, all $s \in S$ and all φ,$\psi \in \mathcal{L}_{upopal}$,

For $N^{\subseteq\psi}$,

- No reduction theorems for $[\psi]O\varphi$ in $\mathcal{L}_{upopal}$.
  - Assume there is a reduction theorem $[\psi]O\varphi \leftrightarrow \chi$, where χ contains no subformulas has $[\alpha]O\beta$ forms. With Proposition 9, we can prove that $\mathcal{L}_{upopal-[\ ]}$ is expressively equal to $\mathcal{L}_{upopal}$ in $N^{\subseteq\psi}$ semantics. Consider a model $\mathcal{M}$, where $S = \{w,u,v\}$, $V(p) = \{w\}$, $V(q) = \{w,u\}$ $N(w) = \{\{w\},\{w,u,v\}\}$. From semantics definition, we have $\mathcal{M}, w \vDash [q]Op$. Let $\mathcal{M}' = \langle S, V, N'\rangle$, where $N'(w) = \{\{w\},\{w,u\},\{w,u,v\}\}$. From Proposition 7, for any $\varphi \in \mathcal{L}_{upopal-[\ ]}$, $\mathcal{M}, w \vDash \varphi$ iff $\mathcal{M}', w \vDash \varphi$. Using $[\psi]O\varphi \leftrightarrow \chi$ and other reduction theorems, we can find a formula χ_1, $\chi_1 \in \mathcal{L}_{upopal-[\ ]}$ and $\vDash \chi_1 \leftrightarrow [q]Op$. So, $\mathcal{M}', w \vDash [q]Op$, but it does not. So, we get a contradiction. Therefore, there is no reduction theorem for $[\psi]O\varphi$ in $\mathcal{L}_{upopal}$.

For $N^{\Cap\psi}$,

- No reduction theorems for $[\psi]P\varphi$ in $\mathcal{L}_{upopal}$.
  - Assume there is a reduction theorem $[\psi]P\varphi \leftrightarrow \chi$, where χ contains no subformulas has $[\alpha]P\beta$ forms. With Proposition 9, we can prove that $\mathcal{L}_{upopal-[\ ]}$ is expressively equal to $\mathcal{L}_{upopal}$ in $N^{\Cap\psi}$ semantics. Consider a model $\mathcal{M}$, $S = \{w,u,v\}$, $V(q) = \{u,v\}$, $V(p) = \{w,u\}$, $N(w) = \{\{w\},\{w,u,v\}\}$. We have $\mathcal{M}, w \vDash \neg[q]p$. Let $\mathcal{M}' = \langle S, V, N'\rangle$, where $N'(w) = \{\{w\},\{w,u\},\{w,u,v\}\}$. Same as $N^{\subseteq\psi}$, we can find a formula χ_1, $\chi_1 \in \mathcal{L}_{upopal-[\ ]}$ and $\vDash \chi_1 \leftrightarrow \neg[q]Pp$ and for all formulas in $\mathcal{L}_{upopal-[\ ]}$, $\mathcal{M}, w \vDash \varphi$ iff $\mathcal{M}', w \vDash \varphi$. So, $\mathcal{M}', w \vDash \neg[q]Pp$, which is not. Therefore, there is no reduction theorems for $[\psi]P\varphi$ in $\mathcal{L}_{upopal}$.

These results tell us that if we want complete reduction theorems for both P and O, we can not only use one of these two update methods. For the proof of completeness, a modified update method is, for $[\psi]P\varphi$ formulas, we use $\subseteq$ semantics, for $[\psi]P\varphi$ formulas, we use $\Cap$ semantics:

Definition 4. *Let* $\mathcal{M}$ *be a model and* s *be a state of* S*, for all* $\varphi, \psi \in \mathcal{L}_{upopal}$*, we define:*

- $\mathcal{M}, s \vDash [\psi]\varphi$ *iff* $\mathcal{M}, s \vDash \psi \Rightarrow \mathcal{M}^{\subseteq\Cap\psi}, s \vDash \varphi$.

If

- $\varphi = P\chi$, $\mathcal{M}^{\subseteq\Cap\psi}, s \vDash \varphi$ *iff* $\mathcal{M}^{\subseteq\psi}, s \vDash \varphi$;
- $\varphi = O\chi$, $\mathcal{M}^{\subseteq\Cap\psi}, s \vDash \varphi$ *iff* $\mathcal{M}^{\Cap\psi}, s \vDash \varphi$;

- *otherwise,* $\mathcal{M}^{\subseteq \Cap \psi}, s \vDash \varphi$ *iff* $\mathcal{M}^{\Cap \psi}, s \vDash \varphi$ *or* $\mathcal{M}^{\subseteq \psi}, s \vDash \varphi$.

Using these definition, we translate any formulas φ in $\mathcal{L}_{upopal}$ to formulas $tr(\varphi)$ without operator [ ]:

Definition 5. *We define* $tr(\varphi)$ *by induction on the complexity of* φ *as follows:*

$tr(p) = p$	$tr(\bot) = \bot$	$tr(\neg\varphi) = \neg tr(\varphi)$
$tr(\psi \vee \varphi) = tr(\psi) \vee tr(\varphi)$	$tr(K_i\varphi) = K_i(tr(\varphi))$	$tr(\varphi) = P(tr(\varphi))$
$tr(\psi \vee \varphi) = tr(\psi) \vee tr(\varphi)$	$tr(K_i\varphi) = K_i(tr(\varphi))$	$tr([\psi][\chi]\varphi) = tr([\langle\psi\rangle\chi]\varphi)$
$tr([\psi]P\varphi) = tr(\psi) \to P(tr(\langle\psi\rangle\varphi))$	$tr([\psi]O\varphi) = tr(\psi) \to O(tr([\psi]\varphi))$	

We have:

Proposition 7. *For all* $\varphi \in \mathcal{L}_{upopal}$, $\vDash \varphi \leftrightarrow tr(\varphi)$

$tr(\varphi)$ is a formula in $\mathcal{L}_{upopal-[\,]}$, which means $\mathcal{L}_{upopal}$ and $\mathcal{L}_{upopal-[\,]}$ is expressively equivalent. Because $\mathcal{L}_{upopal-[\,]}$ is a fragment of $\mathcal{L}_{upopal}$, and using Proposition 7 we can get every formulas in $\mathcal{L}_{upopal}$ is equal to a $\mathcal{L}_{upopal-[\,]}$ formula in semantics.

4 Soundness and Completeness of UPOPAL

4.1 The Axiomatization UPOPAL

In this part, we define the axiomatization UPOPAL. We first give some important valid formulas and validity preserved rules in $\mathcal{L}_{upopla}$.

Proposition 8. *For all models* $\mathcal{M}$ *and all formulas* $\varphi, \varphi_1, \varphi_2 \in \mathcal{L}_{akn}$

- $\vDash O\varphi_1 \wedge O\varphi_2 \to O(\varphi_1 \wedge \varphi_2)$
- $\vDash P\varphi_1 \wedge O\varphi_2 \to P(\varphi_1 \wedge \varphi_2)$
- $\vDash \neg O\varphi \to P\top$

Proposition 9. *For all models* $\mathcal{M}$ *and all formulas* $\varphi, \varphi' \in \mathcal{L}_{akn}$*: if* $\mathcal{M} \vDash \varphi \to \varphi'$*, then* $\mathcal{M} \vDash \varphi \to \varphi'$ *and* $\mathcal{M} \vDash O\varphi \to O\varphi'$

Proof. In [1] Balbiani and Seban proved these formulas' validity on $P(\psi, \varphi)$ and $O(\psi, \varphi)$. We have $P\varphi := P(\top, \varphi)$, $O\varphi := O(\top, \varphi)$, our proof is given straight.

Let UPOPAL be the least set of formulas in $\mathcal{L}_{upopal}$ that contains the axiom schemata and is closed under those inference rules. (See Definition 6.) We define $\vdash_{upopal} \psi$ iff $\psi \in UPOPAL$, and $\Sigma \vdash_{upopal} \varphi$ iff there is a finite set of formulas $\{\chi_1, ...\chi_n\} \subseteq \Sigma$, such that $\vdash_{upopal} (\chi_1 \wedge ... \wedge \chi_n) \to \psi$. A set of formulas Σ is UPOPAL-consistent iff $\Sigma \nvdash_{upopal} \bot$. A formulas set s is a maximal UPOPAL-consistent set iff s is UPOPAL-consistent and for all formulas φ, $\varphi \in s$ or $\neg\varphi \in s$.

Definition 6. *The logic UPOPAL is axiomatized as follows:*

1. *All propositional tautologies*
2. *Theorems of S5 (K,T,B,4)*
3. *Reduction Theorems in Propositions 8 and 9*
4. $O\top$
5. $\neg P\bot$
6. $O\varphi \wedge O\psi \rightarrow O(\varphi \wedge \psi)$
7. $P\varphi \wedge O\psi \rightarrow P(\varphi \wedge \psi)$
8. $\neg O\varphi \rightarrow P(\top)$

R1 *From* φ *and* $\varphi \rightarrow \psi$ *infer* ψ
R2 *From* φ *infer* $K_i\varphi$
R3 *From* φ *infer* $[\psi]\varphi$
R4 *From* $\varphi \rightarrow \varphi'$ *infer* $P\varphi \rightarrow P\varphi'$ *and* $O\varphi \rightarrow O\varphi'$

4.2 Soundness

Proof. The soundness of Axioms 4–8, R4 is from Propositions 7, 8, and 9. Others are standard. See [10] for details.

Then we have:

Proposition 10. *For all* $\varphi \in \mathcal{L}_{upopal}, \vdash_{popal} \varphi \leftrightarrow tr(\varphi)$.

4.3 Completeness

The canonical model is defined below:

Definition 7. *The canonical model* $\mathcal{M}^c = (S^c, \sim^c, V^c, \mathcal{P}^c)$ *can be defined as:*

- S^c *is the set of all maximal UPOPAL-consistent sets*
- *for any* $p \in \Theta, V^c(p) = \{x \in S | p \in x\}$
- $x \sim_i^c y$ *iff* $K_i x = K_i y$*, where* $K_i x = \{\varphi | K_i\varphi \in x\}$
- $N^c(x) = \{S : \exists\varphi(P\varphi \in x \wedge S = \{y \in S^c : \varphi \in y\} \cap \{y \in S^c : \forall O\chi \in x, \chi \in y\})\}$

Proposition 11. *If* $S \in N^c(s)$*, then* S *is not an empty set. (i.e.* $\mathcal{M}^c$ *is a model.)*

Proof. Consider an arbitrary $S_1 \in N^c(s)$. Let $P\varphi_1 \in s_1$ and $S_1 = \{y \in S^c : \varphi \in y\} \cap \{y \in S^c : \forall O\chi \in s_1, \chi \in y\}$. We should prove that $\{\varphi\} \cup \{\chi | \forall\chi \in s_1\}$ is consistent. For contradiction, assume that $\{\varphi\} \cup \{\chi | \forall\chi \in s_1\}$ is not consistent, then we have a finite sequence that $\{\varphi\} \cup \{\chi_1 \wedge \chi_2 ... \wedge \chi_n\} \vdash \bot$, which means $\vdash \varphi \wedge \chi_1 \wedge \chi_2 ... \wedge \chi_n \rightarrow \bot$. From R4, we have $\vdash P(\varphi \wedge \chi_1 \wedge \chi_2 ... \wedge \chi_n) \rightarrow P\bot$. From the condition, we have $O\chi_1 \wedge O\chi_2 \wedge O\chi_3 ... \wedge O\chi_n \in s_1$ and $\varphi \in s_1$, from axiom 6 and axiom 7 we have $P(\varphi \wedge \chi_1 \wedge \chi_2 ... \wedge \chi_n) \in s_1$, then we have $P\bot \in s_1$. But we also have $\neg P\bot \in s_1$ because $\neg P\bot$ is an axiom. This is a contradiction. Therefore, $\{\varphi\} \cup \{\chi | \forall\chi \in s_1\}$ is consistent and S is not empty.

Proposition 12. *(Truth Lemma for $\mathcal{L}_{upopal}$) For all $\varphi \in \mathcal{L}_{upopal}$ we have:*

- *For all $x \in S^c$, $M^c, x \vDash \varphi$ iff $\varphi \in x$*

Proof. The proof is inductive on the *deg* of formulas by using Propositions 6 and 4. Here we only consider non-basic cases of $O\varphi$ and $P\varphi$. See [1] for other steps:

- $\varphi = P\varphi$
  - ($\Longrightarrow$) Suppose that $\mathcal{M}^c, x \vDash P\varphi$, then we get $S_1 \subseteq [\![\varphi]\!]_{\mathcal{M}^c}$, $S_1 \in N^c(x)$. From definition, exists $P\alpha$, $S_1 = \{y : \alpha \in y\} \cap \{y : \forall O\chi \in x, \chi \in y\}$. Now we have $\{y : \alpha \in y\} \cap \{y : \forall O\chi \in x, \chi \in y\} \subseteq [\![\varphi]\!]^c_{\mathcal{M}}$. From Proposition 15 we have $\{\alpha\} \cup \{\chi | \forall O\chi \in x\}$ is consistent, and by hypothesis, we have $\{\alpha\} \cup \{\chi | \forall O\chi \in x\} \cup \{\neg\varphi\}$ is not consistent. Thus $\{\alpha\} \cup \{\chi | \forall O\chi \in x\} \vdash \varphi$, then there is a finite sequence satisfy $\{\alpha\} \cup \{\chi_1 \wedge \chi_2 ... \wedge \chi_n\} \vdash \varphi$. So from R4 we have $P(\alpha \wedge \chi_1 \wedge \chi_2 ... \wedge \chi_n) \rightarrow P\varphi$. We have $O\chi_1 \wedge O\chi_2 \wedge O\chi_3 ... O\chi_n \in x$ and $P\alpha \in x$, from axiom 6 and axiom 7 we have $P(\alpha \wedge \chi_1 \wedge \chi_2 ... \wedge \chi_n) \in x$, then we have $P\varphi \in x$.
  - ($\Longleftarrow$) Suppose $P\varphi \in x$. From proposition15 and induction hypothesis, we can conclude that $\{\varphi\} \cup \{\chi | \forall \chi \in x\}$ is consistent and we can find a non-empty set $S = \{y : \varphi \in y\} \cap \{y : \forall O\chi \in x, \chi \in y\}$. Due to the induction hypothesis, $S \subseteq [\![\varphi]\!]_{\mathcal{M}^c}$, $M^c, x \vDash P\varphi$
- $\varphi = O\varphi$
  - ($\Longrightarrow$) Suppose that $\mathcal{M}^c, x \vDash O\varphi$. Assume $O\varphi \notin x$, from Axiom 8, we have $P\top \in x$. Thus exist $(s, S_1) \in \mathcal{P}^c, S_1 = \{y \in S^c : \top \in y\} \cap \{y \in S^c : \forall O\chi \in x, \chi \in y\})\} = \{y \in S^c : \forall O\chi \in x, \chi \in y\}$. From $\mathcal{M}^c, x \vDash O\varphi$, we have $\{y \in S^c : \forall O\chi \in x, \chi \in y\} \subseteq [\![\varphi]\!]_{\mathcal{M}^c}$. From the induction hypothesis, we have $\{y \in S^c : \forall O\chi \in x, \chi \in y\} \subseteq \{y \in S^c : \varphi \in y\}$. Similar to Proposition 16, we can conclude that $\vdash \chi_1 \wedge \chi_2 ... \wedge \chi_n \rightarrow \varphi$. From R4, we have $\vdash O(\chi_1 \wedge \chi_2 ... \wedge \chi_n) \rightarrow O\varphi$, and we have $O(\chi_1 \wedge \chi_2 ... \wedge \chi_n) \in x$, then $O\varphi \in x$. This is a contradiction.
  - ($\Longleftarrow$) Suppose $O\varphi \in x$ and $\mathcal{M}^c, s \nvDash O\varphi$. Then we have a S_1, $S_1 \in N^c(x)$ and $S_1 \nsubseteq [\![\varphi]\!]_{\mathcal{M}^c}$. From the induction hypothesis, $S_1 \nsubseteq \{y \in S^c : \varphi \in y\}$. However, it is impossible because $S_1 \subseteq \{y \in S^c : \varphi \in y\}$ since $O\varphi \in x$. So $\mathcal{M}^c, s \vDash O\varphi$.

Proposition 13. *$\mathcal{L}_{upopal}$ is complete w.r.t. the class of all frames.*

Proof. For all $\varphi \in \mathcal{L}_{upopal}$: $\vDash \varphi \Longrightarrow$ (Proposition 7) $\vDash tr(\varphi) \Longrightarrow$(definition of $\vDash$) $M^c \vDash tr(\varphi) \Longrightarrow$(Using Truth Lemma) $\vdash tr(\varphi) \Longrightarrow$ (Proposition 10)$\vdash \varphi$.

5 Example: Norms of Assertion and Moorean Sentences

5.1 Knowledge and Belief Norms of Assertion

Timothy Williamson famously proposed that knowledge is the norm of assertion. His argument is "How do you know that?" and "You do not know that!" arguments: if a speaker asserts something, one may always ask her how she knows it and if she does not know it, she can be criticized for not knowing it. This phenomenon can be explained by the knowledge norm of assertion. Besides the argument, the knowledge norm of assertion also explains why it is odd to assert Moorean sentences, which is not permitted under the knowledge norm [13].

The formalization of the knowledge norm of assertion has raised a lot of discussions on the epistemic norms of assertion. (The discussion can be found in [9][4][8].) In this section, we proposed a way to formalize knowledge and belief norms of assertion in our language $\mathcal{L}_{upopal}$, and we also show under the knowledge norm, Moorean sentences are never permitted to announce.

Before we propose our formalization of the knowledge norm, we should make some clarification: (1) We only consider the single-agent situation, so we use $\sim$ instead of $\sim_i$, K instead of K_i. (2) We presuppose the announcement is asserted by the only agent. Intuitively, the knowledge norms of assertion can be regarded as:

- (Knowledge Norm) If agent i is permitted to announce φ, then i knows φ.

We can formalize this as:

- (KN) $P\varphi \to K\varphi$, where φ is a formula in $\mathcal{L}_{upopal}$

From a semantics perspective:

Proposition 14. *KN define the property that for all $S' \subseteq S$ and all $s \in S$, if $S' \in N(s)$, then $\emptyset \neq [s]_\sim \subseteq S'$, where $[s]_\sim$ is the equivalence class of $\sim$ on s.*

Proof. From the property to KN, let $\mathcal{M}, s_1 \vDash P\varphi$, then exists $S_1 \in N(s_1)$, $S_1 \subseteq [\![\varphi]\!]_\mathcal{M}$. From our property, $[s]_\sim \subseteq S_1$, so $[s]_\sim \subseteq [\![\varphi]\!]_\mathcal{M}$. Therefore, we have $\mathcal{M}, s_1 \vDash K\varphi$

From KN to the property, consider the contraposition. In a model $\mathcal{M}$, suppose exist a s_2, $S_2 \in N(s_2)$ and $[s_2]_\sim \not\subseteq S_2$, we can find a $s_3 \in [s_2]_\sim$ but not in S_2. Let $V(p) = S_2$, and $\mathcal{M}, s_2 \vDash Pp \wedge \neg Kp$.

The semantic property shows that the epistemic indistinguishable states of i on s are contained by every ideology, so the agent knows everything not contradicting any ideology. So if someone announces something, and it does not contradict any ideology, then it must not contradict the knowledge cluster of the agent. Therefore, permission implies knowledge.

Knowledge Norms of assertion can easily swift to belief norms BN. To do that, we consider an axiomatization that changes the fragment of epistemic logic $S5$ to belief logic $KD45$. The semantic property will be similar.

5.2 Moorean Sentences

First, we review Moorean sentences:

1. "It is raining, but I do not know that it is raining." (Knowledge version)

2. "It is raining, but I do not believe that it is raining." (Belief version)

We formalize them as $\neg K_i p \wedge p$ and $\neg B_i p \wedge p$. By using KN and BN, we can prove that Moorean sentences are never permitted to announce:

Proposition 15. *In UPOPAL, $\neg P(\neg K_i p \wedge p)$ is valid by adding KN. In UPOPALB, which changes the fragment $S5$ of UPOPAL to belief logic $KD45$, $\neg P(\neg B_i p \wedge p)$ is valid by adding BN.*

Proof. Assume $P(p \wedge \neg Kp)$ in UPOPAL+KN, $P(p \wedge \neg Bp)$ in UPOPALB+BN:

Knowledge version

1.$P(p \wedge \neg Kp)$	Assumption
2.$\vdash p \wedge \neg Kp \to p$	Tautology
3.$\vdash p \wedge \neg Kp \to \neg Kp$	Tautology
4. $\vdash P(p \wedge \neg Kp) \to P(\neg Kp)$	2, R4
5. $\vdash P(p \wedge \neg Kp) \to Pp$	3, R4
6. $\vdash P(\neg Kp) \wedge Pp$	1,4,5 M. P.
7. $\vdash P(\neg Kp) \to K\neg Kp$	KN
8. $\vdash Pp \to Kp$	KN
9. $\vdash K(\neg Kp) \wedge Kp$	6,7,9 M. P.
10. $\vdash K(\neg Kp) \to \neg Kp$	T
11. $\vdash \bot$	9,10 M. P.

Belief version

1. $P(p \wedge \neg Bp)$	Assumption
2. $\vdash p \wedge \neg Bp \to p$	Tautology
3. $\vdash p \wedge \neg Bp \to \neg Bp$	Tautology
4. $\vdash P(p \wedge \neg Bp) \to P(\neg Bp)$	2, R4
5. $\vdash P(p \wedge \neg Bp) \to Pp$	3, R4
6. $\vdash P(\neg Bp) \wedge Pp$	1,4,5 M. P.
7. $\vdash P(\neg Bp) \to B\neg Bp$	BN
8. $\vdash Pp \to Bp$	BN
9. $\vdash B\neg Bp \to \neg BBp$	D
10. $\vdash \neg BBp \to \neg Bp$	4
11. $\vdash Bp$	6,8 M. P.
12. $\vdash \neg Bp$	6,7,9,10 M.P.
13. $\bot_{11,12}$	

Therefore, $\neg P(p \wedge \neg Kp)$ is valid in $\mathcal{L}_{upopal} + KN$ and $\neg P(p \wedge \neg Bp)$ is valid in $\mathcal{L}'_{upopal}+BN$. We can conclude that Moorean sentences are always not permitted to announce, which explains their oddities.

6 Conclusion and Further Work

Based on the work of Balbiani and Seban, we investigate $\mathcal{L}_{upopal}$ and established an axiomatization UPOPAL and prove its soundness and completeness. We also used this logic to formalize knowledge norms of assertion, discuss their semantic property and prove Moorean sentences are never permitted to announce in UPOPAL.

A lot of issues need further work, here we list some of them:

1. Besides $\Cap$ and $\subseteq$, it is possible to find another update method that is natural for $\mathcal{L}_{upopal}$.
2. Another way is to try to prove the completeness without reduction theorems, which is also left as further work. A work of completeness proofs on PAL without reduction theorems can be found in [12].
3. What is odd in UPOPAL is that $O\top$ is valid. The oddity comes that it is not always free to announce $\top$. For example, when a teacher asks a pupil to answer "Is 2+3=5 correct?", she is not free to answer "yes or no".

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