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Ignorance and Temporal Prepositions: A new team-based approach*

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Abstract. In this paper, we provide a team-based semantics for temporal prepositions like *by* and *from*. Following Nouwen [12], we distinguish between class A (before/after) and class B (by/from) temporal expressions. We discuss different linguistic data and the inferences that class A and class B temporal expressions generate. In particular, we claim that only class B expressions trigger ignorance effects. We then propose to analyse *by* and *from* as disjunctions (e.g. *by t* is equivalent to *t OR before*). Finally, we develop a novel state-based system – which builds on Aloni [2] – that accounts for the behaviour of class B expressions. The novelty of this approach resides in the use of *team-based* temporal modalities as an alternative semantics for temporal prepositions.

Keywords: Temporal Logic · Team-based Semantics · Ignorance Inferences

1 Introduction

Modified numerals, such as *more than three* or *at least four*, vary in the ignorance inferences they generate. In (1b), the superlative modifier *at least* suggests that the speaker does not know how many sides a hexagon has and deems possible that it may have five or more, resulting in oddity:

(1) a. A hexagon has more than four sides.

b.# A hexagon has at least five sides.

In [2] and [3], Aloni and van Ormondt develop a *Bilateral State-based Modal Logic* (BSML) which links the ignorance component of superlative modifiers to ignorance inferences typically observed in disjunctions: *at least n* is equivalent to *n OR more*.

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In [12], Nouwen generalizes the contrast in (1) to a broader distinction between class A and class B modifiers. Class A modifiers, like comparative modifiers, relate a numeral to some known cardinality. Class B modifiers, like superlative modifiers, relate a numeral to a range of possible values.

In this paper, we extend Nouwen's class A/B distinction to temporal prepositions. Class A temporal expressions, like *before/after*, relate specific instants of time, while class B temporal expressions, like *by/from*, relate an instant of time to some interval[1]. The following example illustrates that temporal expressions give rise to the same effect obtained in (1)[2]:

(2) a. Christmas is celebrated before the end of December.

b.# Christmas is celebrated by the end of December.

By considering different linguistic data, we conclude that only class B temporal expressions trigger ignorance effects. Following Büring [5] and Aloni [2], we analyse expressions like *by* with an inherent disjunction (e.g. *by t* is equivalent to *t OR before*). We then develop a formal framework which builds on Aloni's team-based BSML [2], extended with temporal modalities. The resulting system, BSTL, is able to account for the inferences generated by class B temporal expressions. The novelty of our approach resides in the use of *team-based* temporal modalities as an alternative semantics for temporal prepositions.

This paper is structured as follows. In Section 2, we present the data and our desiderata. In Section 3, we outline the basics of BSML, extend BSML to *Bilateral State-based Temporal Logic* (BSTL) and prove our results. Section 4 concludes.

2 The Data and Hypothesis

In this section, we discuss the different inferences that class A and class B temporal expressions generate. We then propose to analyse the latter as disjunctions in order to account for their behaviour.

[1] Note that, however, this is not the only use of *by*. As a reviewer suggested, sometimes a *by*-phrase can specify when a process ends, as in "The rocket was ready for takeoff by 4:45 precisely". In the present paper, we focus on more common uses of *by*: those which involve situations that can occur in a bounded interval of time, instead of a specific instant. We speculate that in cases as the one above, a cooperative speaker would prefer the preposition *at* in accordance to the Gricean Maxim of Quantity.

[2] (2a) could also give rise to an ignorance inference: the speaker doesn't know when Christmas is; if they did, they would give the exact date. Thanks to one referee for pressing this point. However, notice that the speaker could also utter this sentence in a situation of full knowledge: say, if the end of December is contextually relevant. On the other hand, (2b) always yields an ignorance inference.

2.1 Ignorance

Class A and class B temporal prepositions contrast with respect to the inferences they give rise to in conversation. Only class B expressions generate ignorance effects, while class A expressions are compatible with full knowledge from the speaker. Consider the following example. In the Netherlands, King's Day is celebrated on the 27th of April. A Dutch citizen utters the following sentences:

(3) a. The King's Day takes place before the 1st of May.

b.# The King's Day takes place by the 1st of May.

The first sentence is acceptable, while the second one is not, for (3b) implies that the speaker deems possible that King's Day is on the 1st of May (or before)[3]. It follows that *by* – a class B temporal expression – gives rise to an ignorance inference, while *before* does not.

2.2 Interaction with modals

Consider the following sentence containing a class A temporal expression uttered by the speaker to their friends:

(4) You can come to the party after 5pm.

This sentence can be interpreted in two ways. A stricter reading would be the following: the friends shouldn't come to the party before the given time; namely 5pm. However, a weaker reading is available; namely that the friends can come to the party after 5pm, at 5pm, or ever earlier. In fact, we can see that this weaker reading is possible if we consider:

(5) You can come to the party after 5pm. But you can also come at 4pm.

Following Nouwen's discussion on modified numerals [12], we observe that these two interpretations are not available if we consider class B temporal expressions:

(6) You can come to the party from 5pm. # But you can also come at 4pm.

By announcing (6), the speaker states that there is a temporal bound: 5pm. Thus, it would be infelicitous to add the possibility of coming to the party

[3] (3b), if uttered out of the blue, could also imply that the date for King's Day is not fixed, and April 30 is the latest allowable date to celebrate it. However, given the context provided, we suppose that the speaker knows that King's Day is fixed in the Netherlands. This previous reading would be natural in a sentence like:

(3b)' Easter takes place by the end of April.

Here, the date for Easter is not fixed and it may fall on a different Sunday every year, ranging from March to April. Our team-based approach easily handles cases like this.

earlier[4]. Following Hackle [9] and Nouwen [12], we can explain the ambiguity of (4) by appealing to the semantic scope of the modal and the temporal expression. The strict reading corresponds to the temporal expression taking wide scope over the modal, while the weak reading corresponds to the opposite configuration:

(4) You can come to the party after 5pm.
$\rightsquigarrow$ **Strict reading**: $AFTER$(5pm)[$\Diamond$[come to the party]]
$\rightsquigarrow$ **Weak reading**: $\Diamond$[$AFTER$(5pm)[come to the party]]

A symmetrical case happens when temporal expressions interact with deontic universal modalities. Consider the following sentences:

(7) a. John has to be home before midnight.
b. John has to be home by midnight.

(7b) allows for two readings. An *authoritative* reading – John is not allowed to be home later than midnight – and the so-called *speaker insecurity* reading (discussed by Büring in [5]) – the speaker does not know exactly what is allowed, but knows that John has to be home at a time which is midnight at the latest. On the other hand, (7a) only allows for an authoritative reading: the speaker states that there is a strict temporal bound.
The authoritative reading of (7b) corresponds to a configuration where the modal scopes over the temporal preposition, while the speaker insecurity reading corresponds to the opposite configuration:

(7b) You can come to the party after 5pm.
$\rightsquigarrow$ **Authoritative reading**: $\Box$[BY(Midnight)[John be home]]
$\rightsquigarrow$ **Speaker insecurity reading**: BY(Midnight)[$\Box$[John be home]]

2.3 A disjunctive proposal

Following Büring [5] and Aloni [2], we propose a new disjunctive meaning for class B temporal expressions. We here focus on the prepositions *by* and *from*, which we interpret as follows: 'by t' ≡ '$t \vee$ before' and 'from t' ≡ '$t \vee$ after'. As an example, consider:

[4] Following [12], we treat class B temporal expressions uniformly. However, a reviewer pointed out that empirical studies suggest that superlative modifiers, a subset of class B modified numerals, should not be treated uniformly (see [6], [11]). In particular, these suggest that *at least* – lower-bound superlative modifier – and *at most* – upper-bound superlative modifier – behave differently in interaction with modals. In line with these findings, one could argue that *by* and *from* should not be given a symmetrical treatment. Note in fact that the sentence

(6)' You can come to the party by 5pm.

does not seem to give rise to such strict reading. Due to lack of space, however, we put this issue aside and leave it for future research.

(8) I will cook dinner by 9.
$\rightsquigarrow$ I will cook dinner at 9 OR before 9.

Given our disjunctive approach to temporal B expressions, we also predict *free choice* phenomena. As [2] points out, conjunctive meanings are often derived from disjunctive modal sentences, contrary to the prescriptions of classical modal logic. $\Diamond$-free choice inferences involve deontic or epistemic existential modal verbs in interaction with disjunctions: $\Diamond(\phi \vee \psi) \rightsquigarrow \Diamond\phi \wedge \Diamond\psi$. Similar inferences are observable also in the temporal domain. Here we consider the deontic case:

(9) You may arrive to the party from 6pm.
$\rightsquigarrow$ You may arrive to the party at 6pm OR after 6pm.
$\rightsquigarrow$ You may arrive to the party at 6pm and you may arrive after 6pm.

As Fox observed in [7], disjunctive sentences within the scope of a universal quantifier yield distribution effects. Sentences with disjunctions in the scope of a universal temporal modality show similar distribution inferences[5], as expected:

(10) The postman always used to deliver our mail by 5 pm.

a. $\rightsquigarrow$ The postman sometimes delivered our mail at 5pm and the postman sometimes delivered our mail before 5pm.
$H(\text{5pm} \vee \text{before}) \rightsquigarrow P\ \text{5pm} \wedge P\ \text{before}$.

b. $\rightsquigarrow$ The postman sometimes might have delivered our mail at 5pm and the postman sometimes might have delivered our mail before 5pm.
$H(\text{5pm} \vee \text{before}) \rightsquigarrow P \Diamond \text{5pm} \wedge P \Diamond \text{before}$. [6]

c. $\rightsquigarrow$ It might be that the postman sometimes delivered our mail at 5pm and it might be that the postman sometimes delivered our mail before 5pm.
$H(\text{5pm} \vee \text{before}) \rightsquigarrow \Diamond P\ \text{5pm} \wedge \Diamond P\ \text{before}$.

Sentence (10) has three possible readings, which are determined by the epistemic context. In (10a) – *strong reading* – the speaker knows, for each day, the exact time of delivery: say, Monday at 5pm, Tuesday at 3pm, etc. In (10c) – *weak reading* – the speaker is ignorant about the exact time of delivery for every day: they just know that these were always 5pm at the latest. However, there

[5] Another type of distribution inferences are licensed by a universal nominal quantifier. These so-called *variation effects* were first observed by Nouwen in the numeral domain [14] and then discussed in detail, among others, by Alexandropoulou et al. [13]. A reviewer observed that similar inferences would obtain in the temporal domain. Consider:

Everyone was there by 6pm.
$\rightsquigarrow$ Someone was there at 6pm and someone was there before 6pm.

A first-order extension of our system could capture this and similar inferences. Exploring such extension is left for future work.

[6] While [8], [1] and [16] argue that it is not possible to have an epistemic modal scoping under tense, we here follow [15] who claims the opposite.

is another possible reading, (10b), which we call *mixed reading*. This obtains in a situation in which the speaker knows the exact time of delivery only relative to certain days, but not all: the speaker knew that on Monday the post was delivered at 5pm, but ignored the time of delivery on Tuesday. Despite the similarities between the *weak* and *mixed* reading, there is a key difference: (10b) expresses certain facts about past possibilities, namely, that there was at least one day when the speaker deemed possible that the mail was delivered at 5pm and one in which they deemed possible that the mail was delivered before 5pm. Note that this reading is compatible with a situation in which the speaker has full knowledge at the time of utterance, but didn't in the past. In contrast, (10c) expresses a live possibility at the time of utterance: the speaker *still* ignores the times of delivery.

2.4 Summary of the observed phenomena

Following the previous discussion, we summarize the inferences that our formal system aims to capture. In the following, read H as 'in all instants in the past' and P as 'there is an instant in the past'.

(11) a. Mum will arrive by tomorrow. [Ignorance]

b. (tomorrow $\vee$ before) $\rightsquigarrow \Diamond$ tomorrow $\wedge \Diamond$ before.

(12) a. You may arrive to the party from 6pm. [$\Diamond$-Free Choice]

b. $\Diamond$(6pm $\vee$ after) $\rightsquigarrow \Diamond$ 6pm $\wedge \Diamond$ after.

(13) a. The postman always used to deliver our mail by 5 pm. [Strong Distribution]

b. H(5pm $\vee$ before) $\rightsquigarrow P$ 5pm $\wedge P$ before.

(14) a. The postman always used to deliver our mail by 5 pm. [Mixed Distribution]

b. H(5pm $\vee$ before) $\rightsquigarrow P \Diamond$ 5pm $\wedge P \Diamond$ before.

(15) a. The postman always used to deliver our mail by 5 pm. [Weak Distribution]

b. H(5pm $\vee$ before) $\rightsquigarrow \Diamond P$ 5pm $\wedge \Diamond P$ before.

3 Bilateral State-Based Temporal Logic

In this section, we develop a state-based system, BSTL, that allows us to derive the discussed inferences in a rigorous manner. This is obtained by enriching the framework developed in [2] with novel temporal state-based modalities.

3.1 BSML

In basic modal logic, formulas are interpreted with respect to a possible world, while in team-based modal logic formulas are interpreted with respect to sets of possible worlds. In [2], Aloni develops a bilateral version of team-based modal logic – *Bilateral state-based modal logic* (BSML) – which includes both support and anti-support conditions. Thus, BSML does not model truth in a possible world, but "*assertion* and *rejection* conditions in an *information state*" [2].

We now state the syntax and semantics for BSML. Notice that BSML is characterized by a novel element – the non-emptiness atom NE.

Definition 1. (Language of BSML) Let $\mathcal{A}$ be a countable set of *propositional atoms*, $\mathcal{A} = \{p, q, r...\}$. The language of BSML is recursively defined as follows:

$$p \in \mathcal{A} \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid \Diamond \phi \mid NE$$

In the following, $\mathbb{M}, s \vDash \phi$ reads 'formula ϕ is assertable in s' and similarly $\mathbb{M}, s \dashv \phi$ reads 'formula ϕ is rejectable in s', where $s \subseteq W$ is an information state and M is a Kripke model. For any possible world w, we denote the set of R-successors of w as $R[w] = \{v \in W \mid wRv\}$.

Definition 2. (Semantic clauses for BSML)

$\mathbb{M}, s \vDash p \Leftrightarrow \forall w \in s,\ w \in V(p)$
$\mathbb{M}, s \vDash \neg\phi \Leftrightarrow \mathbb{M}, s \dashv \phi$
$\mathbb{M}, s \vDash \phi \vee \psi \Leftrightarrow \exists t,\ t' \subseteq W$ s.t. $s = t \cup t'$, $\mathbb{M}, t \vDash \phi$ and $\mathbb{M}, t' \vDash \psi$
$\mathbb{M}, s \vDash \phi \wedge \psi \Leftrightarrow \mathbb{M}, s \vDash \phi$ and $\mathbb{M}, s \vDash \psi$
$\mathbb{M}, s \vDash NE \Leftrightarrow s \neq \varnothing$
$\mathbb{M}, s \vDash \Diamond\phi \Leftrightarrow \forall w \in s,\ \exists t \subseteq\ R[w]$ s.t. $t \neq \varnothing$ and $\mathbb{M}, t \vDash \phi$

$\mathbb{M}, s \dashv p \Leftrightarrow \forall w \in s,\ w \notin V(p)$
$\mathbb{M}, s \dashv \neg\phi \Leftrightarrow \mathbb{M}, s \vDash \phi$
$\mathbb{M}, s \dashv \phi \vee \psi \Leftrightarrow \mathbb{M}, s \dashv \phi$ and $\mathbb{M}, s \dashv \psi$
$\mathbb{M}, s \dashv \phi \wedge \psi \Leftrightarrow \exists t,\ t' \subseteq W$ s.t. $s = t \cup t'$, $\mathbb{M}, t \dashv \phi$ and $\mathbb{M}, t' \dashv \psi$
$\mathbb{M}, s \dashv NE \Leftrightarrow s = \varnothing$
$\mathbb{M}, s \dashv \Diamond\phi \Leftrightarrow \forall w \in s,\ \mathbb{M}, R[w] \dashv \phi$

Logical consequence is, as expected, defined as preservation of support in an information state. At last, we introduce a constraint on the accessibility relation R. This will play a crucial role in the proofs of the main results.

Definition 3. Let $\mathbb{M}, s$ be a model-state pair, we say that R is *state-based* in $\mathbb{M}, s \Leftrightarrow \forall w \in s,\ R[w] = s$

Given that a key feature of our proposal is the disjunctive behaviour of class B temporal expression, we now discuss how disjunctions are treated in BSML. BSML adopts a notion of disjunction from dependence logic and team logic known as *split* or *tensor* disjunction [17]. In order to differentiate 'vacuous' disjunctions – i.e. those supported by a state s by means of an empty substate – from 'properly supported' disjunctions – i.e. those supported by means of two nonempty substates – [2] defines recursively a *pragmatic enrichment* function $[]^+$ using the non-emptiness atom NE. This rules out the empty state as a possible substate in the support condition of a pragmatically enriched disjunction:

$$\mathbb{M}, s \vDash [\phi \vee \psi]^+ \Leftrightarrow \exists t,\ t' \subseteq W \text{ s.t. } t, t' \neq \varnothing,\ s = t \cup t',\ \mathbb{M}, t \vDash \phi \text{ and } \mathbb{M}, t' \vDash \psi$$

3.2 BSTL

In order to account for temporal expressions, we need to introduce temporal modalities to BSML. In basic temporal logic, however, satisfaction is defined at world level. If one were to extend BSML with classical temporal modalities, a straightforward way would be to apply the modality to each world within a given state. This, however, would be problematic because it wouldn't allow for actual interaction with epistemic modalities – whose satisfaction conditions are defined at team level. To see this, consider the previous:

a. The postman always used to deliver our mail by 5 pm.
b. $H(\text{5pm} \vee \text{before}) \rightsquigarrow P\Diamond\ \text{5pm} \wedge P\Diamond\ \text{before}$.

A system with world-based satisfaction of temporal modalities cannot deal with an overt epistemic modal scoping under tense: once the (standard) semantic definition of P is applied at each world in a state, there is no direct way in BSML to apply the semantic clause for $\Diamond$ at each of those same worlds. For this reason, we develop a novel framework: *Bilateral state-based temporal logic* (BSTL).

Definition 4. (Language of BSTL) Let $\mathcal{A}$ be a countable set of *propositional atoms*, $\mathcal{A} = \{p, q, r...\}$. The language of BSTL is recursively defined as follows:

$$p \in \mathcal{A} \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid\ \Diamond\,\phi \mid NE \mid P\phi \mid H\phi \mid F\phi \mid G\phi$$

Models of BSTL consist of:

- a set of instants of time W with *root* r,
- two relations on W: a backward-lineal partial order $<$ (representing the temporal precedence) and an accessibility relation R (representing the epistemic indistinguishability between instants), and
- a standard *Kripkean* valuation $V : \mathcal{A} \to \mathcal{P}(W)$.

In order to define the semantic clauses for the temporal modalities we introduce the following key definitions.

Definition 5. (History) Let $(W, <, R, V)$ be a BSTL model and let r be the root of W. Let $D \subseteq \mathbb{N}$ be of the form $\{0, 1...n\}$ or be equal to $\mathbb{N}$. A history is a function $h : D \to W$ such that:

- $h(0)$=r
- $\forall n \in D \setminus \{0\},\ h(n-1) < h(n)$.

Intuitively, h is a (potentially finite) sequence of consecutive instants in W[7]. For any instant $w \in W$, we denote by $\mathcal{H}(w)$ the set of all histories passing through w. For a state s, h_s denotes a choice of histories passing through the instants in s, that is, $h_s = \{h_w \mid h_w \in \mathcal{H}(w)\}_{w \in s}$. We also adopt the following notation: for any natural number k, $h_s(k) = \{h_w(k)\}_{h_w \in h_s}$.
Given this notion of history, we can introduce the concept of *temporal depth* of an instant w: if h describes the minimal[8] path from $r = h(0)$ to $w = h(n)$, then $dep(w) = n$. Given a state $s \subseteq W$, we define the *temporal depth* of s as $dep(s) = min\{dep(w),\ w \in s\}$. Analogously, the *temporal height* of s will be $heg(s) = max\{dep(w),\ w \in s\}$.

Definition 6. (Temporal level) *Temporal levels* are sets of instants with the same temporal depth: the n-th temporal level in W is $l_n = \{w \in W \mid dep(w) = n\}$.

Temporal levels correspond to units in a system that measures the flow of time. For instance, levels can correspond to years, days, hours, etc. Intuitively, the level l_k groups all the epistemically possible instants at the time k. For simplicity, we here assume that information states are *synchronous*: all the instants in a state are at the same temporal level. Recall also that R is the indistinguishability relation. A situation of full epistemic ignorance corresponds to the case where R is state-based at each temporal level. On the other hand, a situation of full knowledge corresponds to the case where, at each temporal level, there are no epistemic arrows between any two different instants. Both cases are illustrated in Fig. 1.

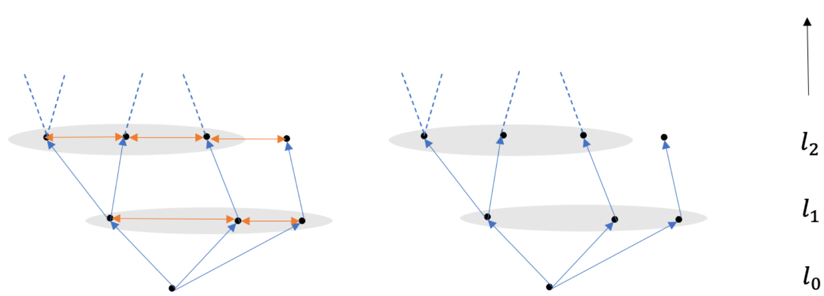

Fig. 1. Examples of intended models of BSTL. The blue arrows represent the flow of time, while the orange arrows represent R. The areas in grey are information states. l_0, l_1, etc. correspond to temporal levels.

[7] We observe here that our notion of history is reminiscent of the concept of *trace* from the team semantics of *linear temporal logic* (LTL). A more detailed discussion can be found in Krebs et al. [10].

[8] That is, $h(0) = r$, $h(n) = w$ and for all other h' with $h'(0) = r$, $h'(m) = w$ we have $n \leq m$.

We now state the semantic clauses for BSTL. BSTL inherits all the semantic clauses of BSML. In addition, we add the satisfaction clauses for P, H, F, G. The rejection conditions are defined in terms of support of the negation, i.e. $\mathbb{M}, s \dashv P\phi \Leftrightarrow \mathbb{M}, s \vDash \neg(P\phi)$ and similarly for the other modalities.

Definition 7. (Semantic clauses for the temporal modalities in BSTL)

$$\mathbb{M}, s \vDash P\phi \Leftrightarrow \exists k \leq dep(s),\ \exists s' \subseteq l_k,\ \exists h_s\ s.t.\ h_s(k) \subseteq s'\ and\ s' \vDash \phi$$
$$\mathbb{M}, s \vDash H\phi \Leftrightarrow \forall k \leq dep(s),\ \exists s' \subseteq l_k\ s.t.\ \forall h_s,\ h_s(k) \subseteq s'\ and\ s' \vDash \phi$$
$$\mathbb{M}, s \vDash F\phi \Leftrightarrow \exists k \geq heg(s),\ \exists s' \subseteq l_k,\ \exists h_s\ s.t.\ h_s(k) \subseteq s'\ and\ s' \vDash \phi$$
$$\mathbb{M}, s \vDash G\phi \Leftrightarrow \forall k \geq heg(s),\ \exists s' \subseteq l_k\ s.t.\ \forall h_s,\ h_s(k) \subseteq s'\ and\ s' \vDash \phi$$

A state s supports $P\phi$ if and only if (a) there is a level $k \leq dep(s)$, (b) there is an information state $s' \subseteq l_k$ supporting ϕ and (c) there is a set of histories that pass through the instants in s such that, if we move backwards along these histories and reach the level k, the resulting state is included in s'. The satisfaction clause for $H\phi$ is analogous to $P\phi$, but applies to all levels $k \leq dep(s)$. Finally, the semantics for $F\phi$ and $G\phi$ are defined in a similar manner.

Now that we have introduced the team-based semantics of our temporal modalities, we are able to interpret sentences like $H \Box \phi$, $P \Diamond \phi$, etc.

3.3 Results:

With our system, we are able to prove all the inferences discussed in 2.4. Due to lack of space, however, we provide complete proofs only of Ignorance and Mixed Distribution.

Ignorance[9]. Let ϕ be a classical formula. Let $\mathbb{M}$ be a BSTL model and $s \subseteq W$ be a state such that R is state-based in $\mathbb{M}, s$. Then we have:

$$[\phi \vee P\phi]^+ \vDash_s \Diamond\phi \wedge \Diamond P\phi$$

Proof. For simplicity, we just consider the atomic case $\phi = p$. The result can be easily generalized to arbitrary ϕ by induction on the complexity of the formula. Let $\mathbb{M}, s \vDash [p \vee Pp]^+$, to show: $\mathbb{M}, s \vDash \Diamond p \wedge \Diamond Pp$. By assumption there exist substates $t, t' \subseteq s$, $t, t' \neq \varnothing$ such that $\mathbb{M}, t \vDash p$ and $\mathbb{M}, t' \vDash Pp$. Since R is state-based, it is also reflexive. By reflexivity of R we have that $\mathbb{M}, t \vDash \Diamond p$ and $\mathbb{M}, t' \vDash \Diamond Pp$. Since $t \subseteq s$, we have $\mathbb{M}, s \vDash \Diamond p$. In fact, let $w \in s$, we need to check that there exists a non-empty substate r of $R[w]$ such that $\mathbb{M}, r \vDash p$. But by state-basedness of R, $R[w] = s$, hence we can take $r = t$. Analogously, since $t' \subseteq s$, we have $\mathbb{M}, s \vDash \Diamond Pp$. Hence, we conclude that $\mathbb{M}, s \vDash \Diamond p \wedge \Diamond Pp$ as desired.

[9] Note that in both results the temporal operators P and H can be uniformly substituted with F and G. For Mixed Distribution, assume R state-based at all levels in the future of s.

Mixed Distribution. Let ϕ be a classical formula. Let $\mathbb{M}$ be a BSTL model and $s \subseteq W$ be a state. Suppose that R is state-based at all temporal levels preceding s. Then we have:

$$[H(\phi \vee P\phi)]^+ \vDash_s P \Diamond \phi \wedge P \Diamond P\phi$$

Proof. Suppose that $\mathbb{M}, s \vDash [H(\phi \vee P\phi)]^+$, to show: $\mathbb{M}, s \vDash P \Diamond \phi$ and $\mathbb{M}, s \vDash P \Diamond P\phi$. Again, we just focus on the atomic case $\phi = p$.
By assumption, we have that $s \neq \varnothing$ and $\mathbb{M}, s \vDash H(p \vee Pp)^+$. By definition, this means that $\forall k \leq dep(s)$, $\exists s' \subseteq l_k$ s.t. $s' \vDash (p \vee Pp)^+$ and $\forall h_s$, $h_s(k) \subseteq s'$. Recall that $\mathbb{M}, s \vDash P \Diamond p \Leftrightarrow \exists k \leq dep(s)$, $\exists s' \subseteq l_k$, $\exists h_s$ s.t. $h_s(k) \subseteq s'$ and $s' \vDash \Diamond p$. Furthermore, $s' \vDash \Diamond p \Leftrightarrow \forall v \in s'$, $\exists t \subseteq R[v]$, $t \neq \varnothing$, such that $t \vDash p$. Similarly, $\mathbb{M}, s \vDash P \Diamond Pp \Leftrightarrow \exists k \leq dep(s)$, $\exists s' \subseteq l_k$, $\exists h_s$ s.t. $h_s(k) \subseteq s'$ and $\forall v \in s'$, $\exists t \subseteq R[v]$, $t \neq \varnothing$, such that $t \vDash Pp$.
Now let $k \leq dep(s)$. By assumption, there exists a state $s' \subseteq l_k$ s.t. for any choice of histories h_s, $h_s(k) \subseteq s'$ and $s' \vDash (p \vee Pp)^+$. Fix one such h_s. Let t_1 and t_2 be the non-empty substates of s' such that $t_1 \vDash p$ and $t_2 \vDash Pp$. Since R is state-based at l_k, this means that for all $v \in s'$, $R[v] = l_k$. Hence we can find nonempty $t_1 \subseteq R[v]$ such that $t_1 \vDash p$ and nonempty $t_2 \subseteq R[v]$ such that $t_2 \vDash Pp$. But this is exactly what we needed to prove.

4 Conclusion and further work

In this paper, we have expanded Nouwen [12]'s class A/class B distinction to the domain of temporal prepositions. We discussed different linguistic data and the inferences that class A and class B temporal expressions generate. We then proposed to analyse the latter as disjunctions. Finally, we developed a novel state-based system that is able to account for the behaviour of class B expressions: BSTL.

However, there is much that remains to be done. A first area for further work is to capture ignorance between different temporal levels. Currently our framework does not account for sentences like:

(16) The postman delivered our mail. But I don't know when.

In this example, it is known that a certain event happened, but the exact time of the event is unknown. This requires the information states to span over different temporal levels, a possibility that was not contemplated in this paper.

Another area for further investigation is the interaction between class B temporal expressions and the lexical aspect of verbs. For example, *atelic* verbs (those verbs which do not involve any goal nor envisaged endpoint – e.g. sleep, walk) seem to block the licensing of a *by*-phrase. Consider:

(17) # Mary slept by 9pm.

Other frameworks, such as Altshuler and Michaelis' [4], are able to account for this fact stipulating that the semantics of *by* requires the existence of a prominent state that is the result of some event. Thus, the oddity of (17) derives from the fact that *slept* is not typically associated with any resultant state. We hypothesize that, in our system, such infelicity could be derived compositionally from the semantic contribution of atelic verbs. In particular, these could impose independent constraints on the states supporting sentences in which they occur.

Furthermore, we would like to investigate whether our system can provide a satisfactory semantics for other temporal prepositions, such as *until* and *since*.

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