

Newcomb's Problem

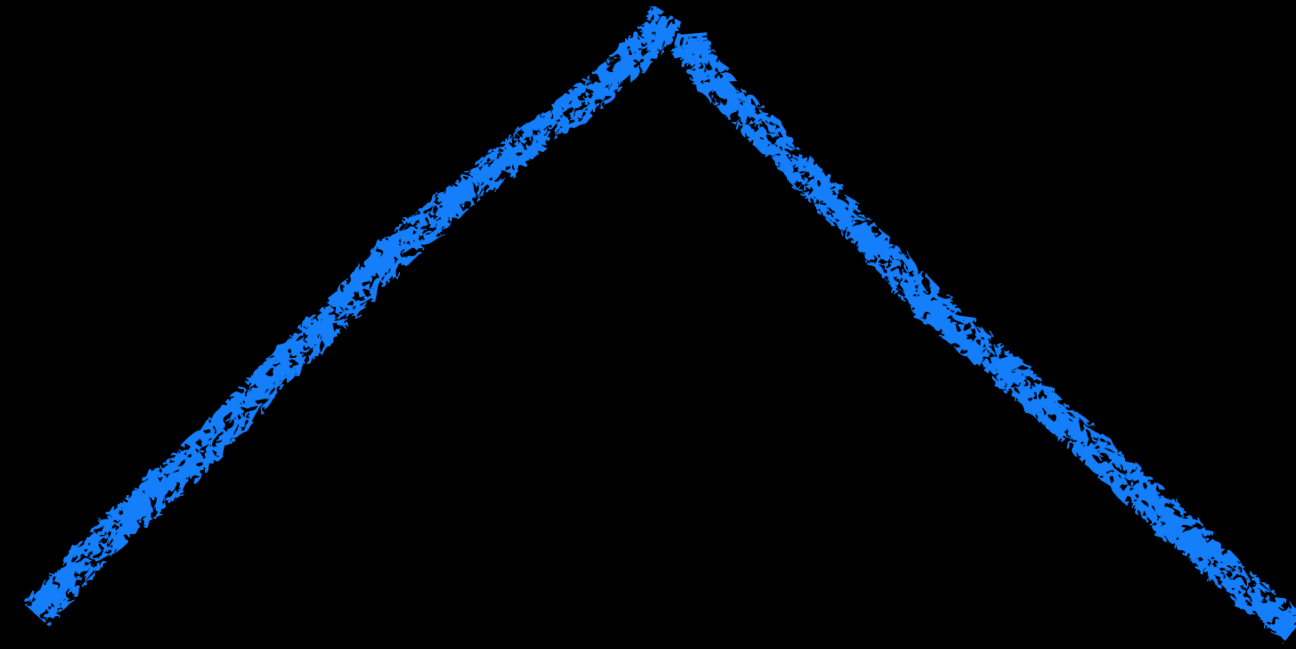
An introduction to

Decision Theory

Φ - Math Reading Group

Friday 29 January 2021

Decision Theory



Descriptive

How do people
actually make
decisions?

Normative

How should
people make
decisions?

Decision Theory

TODAY

Descriptive

How do people
actually
make
decisions?

Normative

How should
people make
decisions?



Heads : + €12
Tails : - €10

Expected Utility

Let:

- O be a set of outcomes
- $u : O \rightarrow \mathbb{R}$ utility
- $p : O \rightarrow [0, 1]$ credences $\leftarrow$ probability distribution
- $A \subseteq O$ actions

$$eu(a) = \sum_{o \in O} u(o) p(o|a)$$



Heads: + €12

Tails: - €10

$O = \{\text{no flip, H, T, play, decline}\}$

$A = \{\text{play, decline}\}$



Heads: +€12

Tails: -€10



$O = \{\text{no flip, H, T, play, decline}\}$

$A = \{\text{play, decline}\}$

$$eu(\text{decline}) = \underset{0}{(0 \cdot 1)} + (+€12 \cdot 0) + (-€10 \cdot 0)$$

$$eu(\text{play}) = \underset{+€1}{(0 \cdot 0)} + (+€12 \cdot .5) + (-€10 \cdot .5)$$



Heads: + €12

Tails: - €10



$O = \{\text{no flip, H, T, play, decline}\}$

$A = \{\text{play, decline}\}$

$$eu(\text{decline}) = \underset{0}{(0 \cdot 1)} + (+€12 \cdot 0) + (-€10 \cdot 0)$$

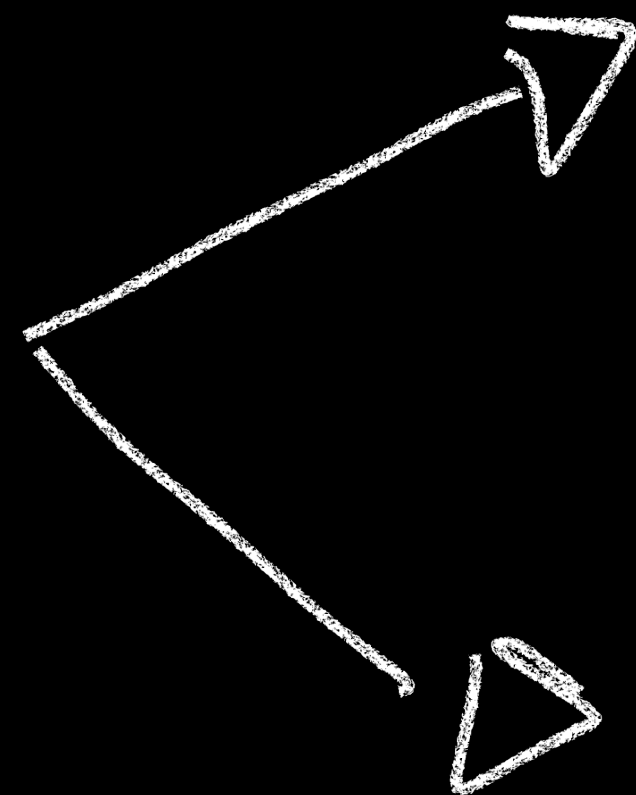
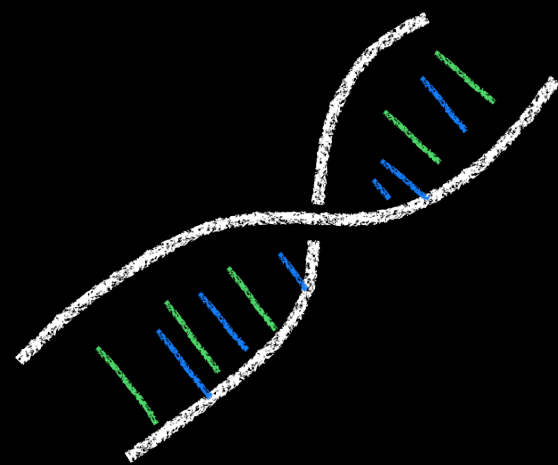
$$eu(\text{play}) = \underset{+€1}{(0 \cdot 0)} + (+€12 \cdot .5) + (-€10 \cdot .5)$$

Recommendation: PLAY!

Doctors:



Smoking Lobby:



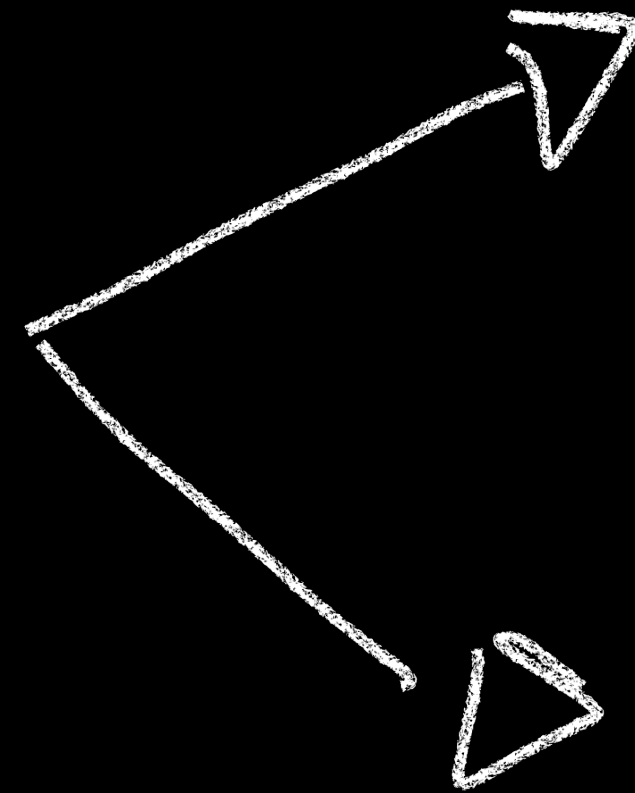
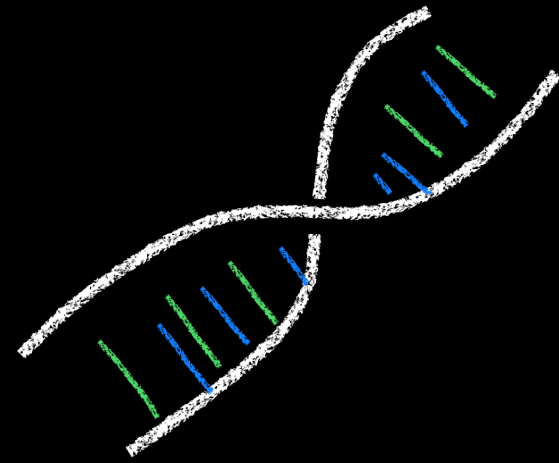
Suppose you enjoy smoking.

Would you smoke if ... the doctors are right?
... the smoking lobby is right?

Doctors :



Smoking Lobby :



Both :



)

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P (



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-



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$$P(\text{👤🔴} \mid \text{🔊})$$

$$= P(\text{👤🔴}, \text{📶} \mid \text{🔊}) + P(\text{👤🔴}, \neg \text{📶} \mid \text{🔊})$$

Law of
total
probability

$$= P(\text{👤🔴} \mid \text{📶}, \text{🔊}) P(\text{📶} \mid \text{🔊})$$

$$+ P(\text{👤🔴} \mid \neg \text{📶}, \text{🔊}) P(\neg \text{📶} \mid \text{🔊})$$

$$= P(\text{👤🔴} \mid \text{📶}) P(\text{📶} \mid \text{🔊})$$

$$+ P(\text{👤🔴} \mid \neg \text{📶}) P(\neg \text{📶} \mid \text{🔊})$$



$$P(\text{Lungs} \mid \text{Cigarette}) = P(\text{Lungs} \mid \text{Smoker}) P(\text{Smoker} \mid \text{Cigarette}) \\ + P(\text{Lungs} \mid \neg \text{Smoker}) P(\neg \text{Smoker} \mid \text{Cigarette})$$

$$P(\text{Lungs} \mid \neg \text{Cigarette}) = P(\text{Lungs} \mid \text{Smoker}) P(\text{Smoker} \mid \neg \text{Cigarette}) \\ + P(\text{Lungs} \mid \neg \text{Smoker}) P(\neg \text{Smoker} \mid \neg \text{Cigarette})$$

$$P(\text{👤🔴} | \text{🚬}) = P(\text{👤🔴} | \text{🧬}) P(\text{🧬} | \text{🚬}) \\ + P(\text{👤🔴} | \neg \text{🧬}) P(\neg \text{🧬} | \text{🚬})$$

$$P(\text{👤🔴} | \neg \text{🚬}) = P(\text{👤🔴} | \text{🧬}) P(\text{🧬} | \neg \text{🚬}) \\ + P(\text{👤🔴} | \neg \text{🧬}) P(\neg \text{🧬} | \neg \text{🚬})$$

Given: $P(\text{🧬} | \text{🚬}) > P(\text{🧬} | \neg \text{🚬})$

$$P(\text{👤🔴} | \text{🧬}) > P(\text{👤🔴} | \neg \text{🧬})$$

$$P(\text{🦠} \mid \text{🚬}) > P(\text{🦠} \mid \neg \text{🚬})$$

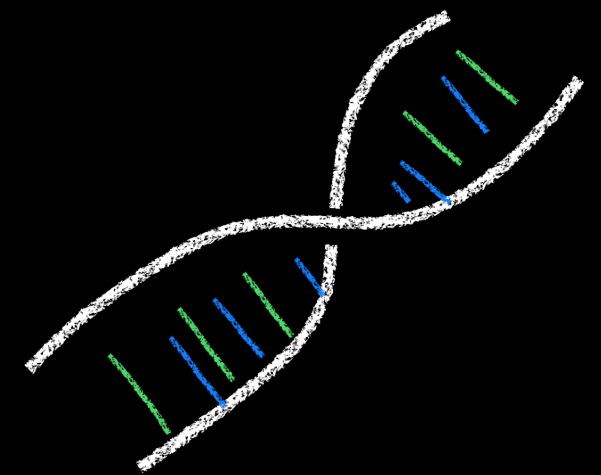
$$\therefore eu(\text{🚬}) < eu(\neg \text{🚬})$$

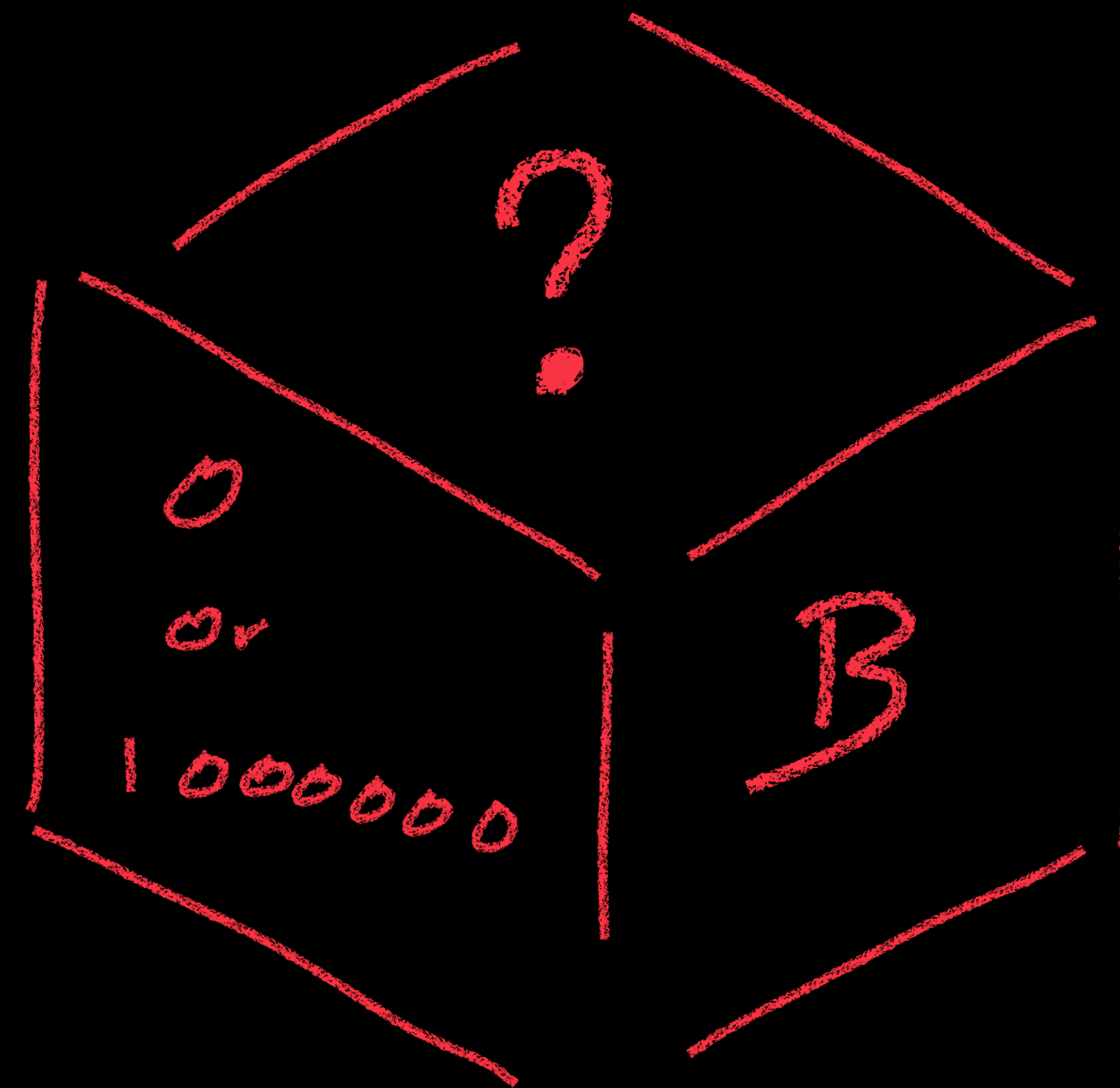
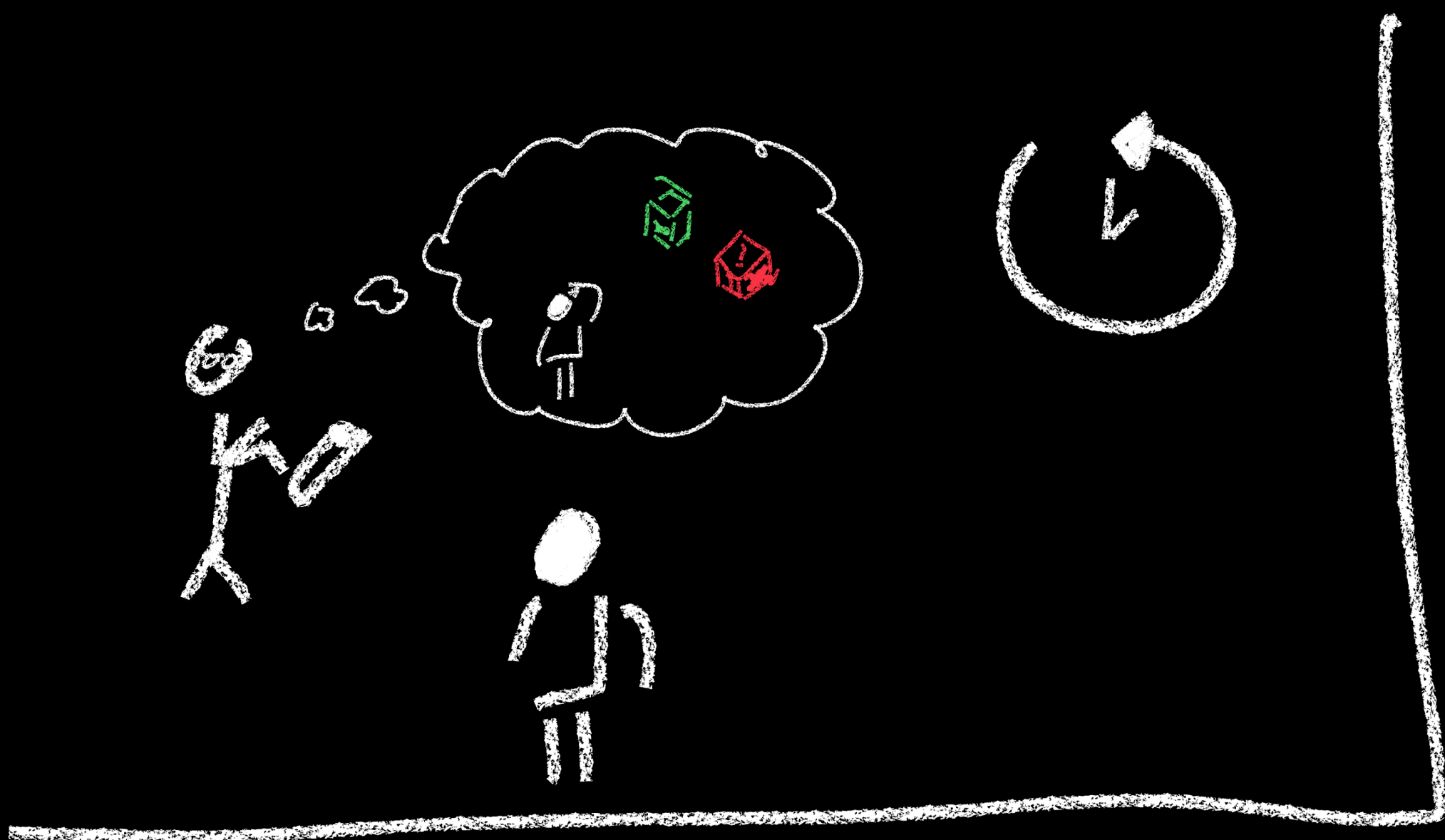
↳ expected utility represents the doctors,
but not the tobacco lobbyists

$$P(\text{DNA} | \text{Microphone}) > P(\text{DNA} | \neg \text{Microphone})$$

$$P(\text{DNA} | \text{Fingerprint}) > P(\text{DNA} | \neg \text{Fingerprint})$$

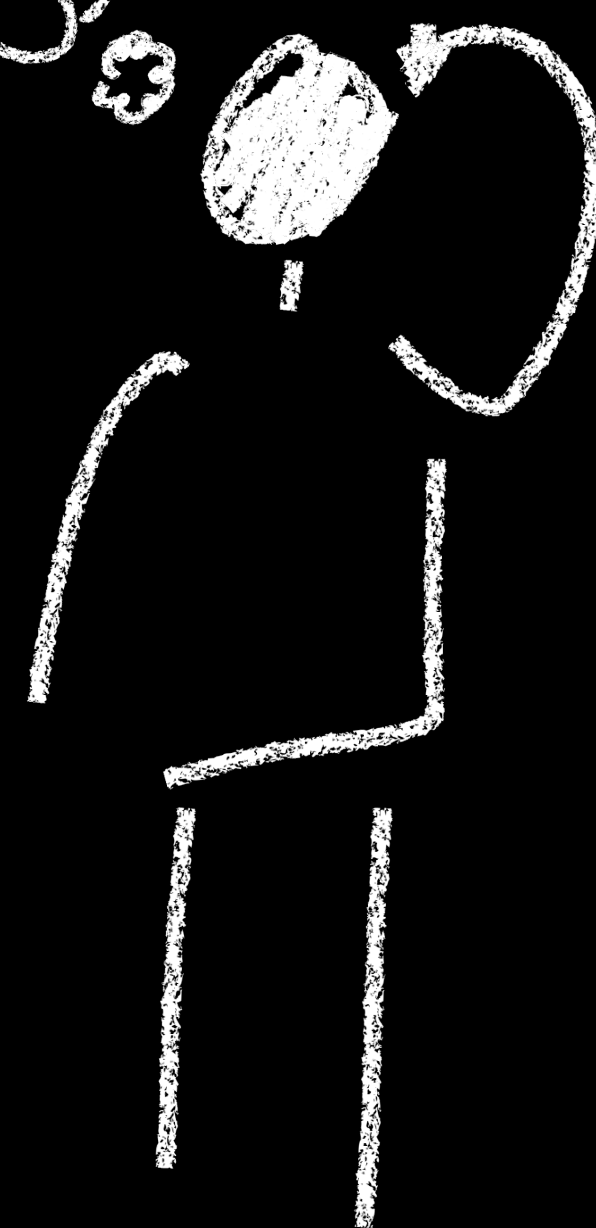
raises one's evidence of
but does not cause

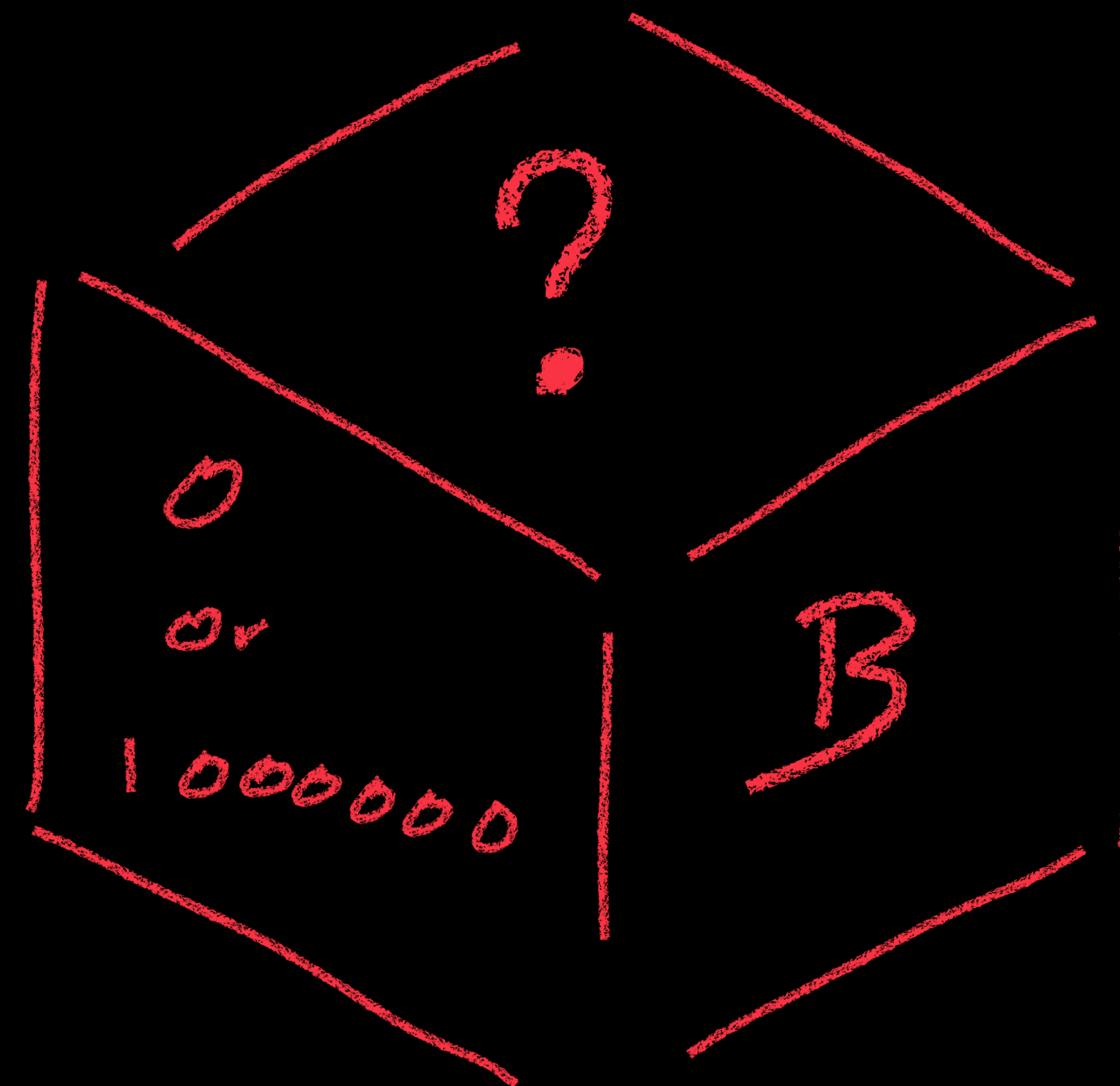
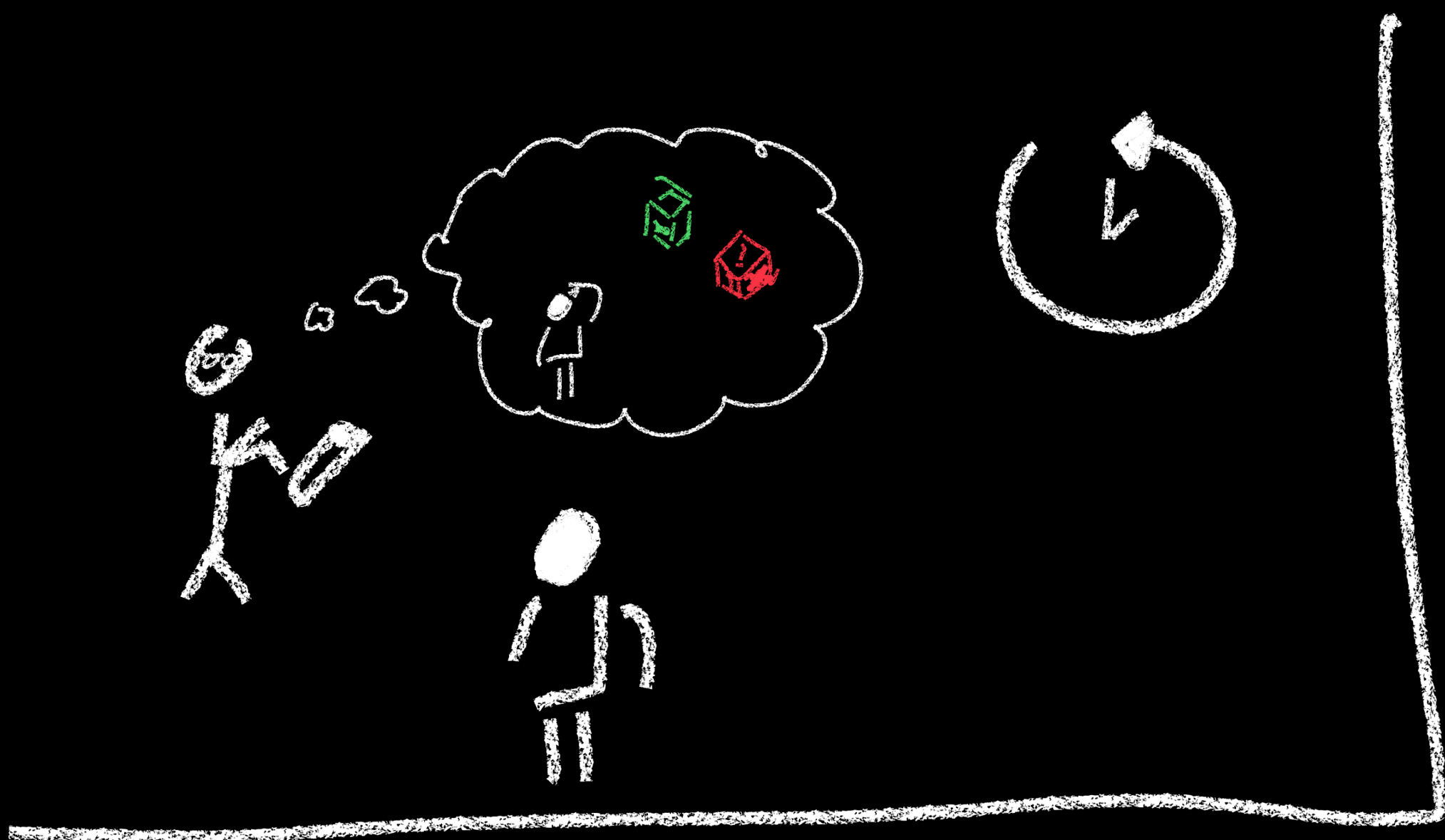




$A \& B \rightarrow B = 0$

$B \rightarrow B = 1000000$





$$A \& B \rightarrow B = 0$$

$$B \rightarrow B = 1000000$$



$$eu(B) > eu(A \& B)$$

What should you do?

What should you do?

Leo Condorandi (2002):

models have a temporal perspective

"They might have won the match."

"At half-time, they might have won."